Hierarchy construction schemes within the Scale set framework

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Abstract. Segmentation algorithms based on an energy minimisation framework often depend on a scale parameter which balances a fit to data and a regularising term. Irregular pyramids are defined as a stack of graphs successively reduced. Within this framework, the scale is often defined implicitly as the height in the pyramid. However, each level of an irregular pyramid can not usually be readily associated to the global optimum of an energy or a global criterion on the base level graph. This last drawback is addressed by the scale set framework designed by Guigues. The methods designed by this author allow to build a hierarchy and to design cuts within this hierarchy which globally minimise an energy. This paper studies the influence of the construction scheme of the initial hierarchy on the resulting optimal cuts. We propose one sequential and one parallel method with two variations within both. Our sequential methods provide partitions near the global optima while parallel methods require less execution times than the sequential method of Guigues even on sequential machines.

1 Introduction

Despite much efforts and significant progresses in recent years, image segmentation remains a notoriously challenging computer vision problem. It's usually a preliminary step towards image interpretation and plays a major role in many applications.

The use of an energy minimisation scheme within the region based segmentation framework allows to define criteria which should be globally optimised over a partition. Several types of methods such as the Level set [1], the Bayesian [2], the minimum description length [3] and the minimal cut [4] frameworks are based on this approach. Within these frameworks the energy of a partition P is usually defined as $E_\lambda(P) = D(P) + \lambda C(P)$ where D and C denote respectively the fit to data and the regularising term. The energy $E_\lambda(P)$ corresponds to the Lagrangian of the constraint problem: minimise $D(P)$ subject to $C(P) \leq \epsilon$. Where ϵ is a function of λ. Under large assumptions, minimising $E_\lambda(P)$ is also equivalent to the dual problem: minimise $C(P)$ subject to $D(P) \leq \epsilon'$, where ϵ' is also a function of λ. Therefore λ may be interpreted as the amount of freedom allowed to minimise D ($D(P) \leq \epsilon'$) while keeping C as low as possible. Since ϵ' is a growing function of λ, as λ is growing, the constraint on D is more and more relaxed while the importance of the term C is getting more and more important. This parameter λ may thus be interpreted as a *scale parameter* which represents the relative weighting between the two energy terms.

In many approaches the parameter λ is fixed experimentally and a minimisation algorithm determines for a value of λ a locally optimal partition from the set $\mathbb{P}$ of all the possible partitions on image I. A sequence of λ may also be defined a priori in order to compute the optimal partition on each sampled value of λ [5].

The scale set framework proposed by Guigues [5] is based on a different approach. Instead of performing the minimisation scheme on the whole set $\mathbb{P}$ of possible partitions of an image I, Guigues proposes to restrict the search on a hierarchy H. The advantages of this approach are twofold: First as shown by Guigues the globally optimal partition on H may be found efficiently while the search on the whole set $\mathbb{P}$ of partitions only provides local minima. Secondly, Guigues shown that if the energy satisfies some basic properties, the whole set of solutions on H when λ describes $\mathbb{R}+$ corresponds to a sequence of increasing cuts within the hierarchy H hereby providing a contiguous representation of the solutions for the parameter λ. A method to build the hierarchy H has been proposed by Guigues. Since the research space used by Guigues is restricted to the initial hierarchy H the construction scheme of this hierarchy is of crucial importance for the optimal partitions *within H* built in the second step.

This paper explores different heuristics to build the initial hierarchy. These heuristics represent different compromises between the energy of the final partition and the execution times. We first present in Section 2 the scale set framework. The different heuristics are then presented in Section 3. These heuristics are evaluated and compared to the method of Guigues in Section 4.

2 The Scale Set framework

Given an image I and two partitions P and Q on I, we will say that P is *finer* than Q (or Q is coarser then P) iff Q may be deduced from P by merging operations. This relationship is denoted by $P \trianglelefteq Q$. Let us now consider a theoretic segmentation algorithm P_λ which returns an optimal partition $P_\lambda(I)$ of I according to E_λ for each value of the scale parameter λ. We will say that P is an *unbiased multi-scale segmentation* algorithm iff for any couple (λ_1, λ_2) such that $\lambda_1 \leq \lambda_2$, and any image I, $P_{\lambda_1}(I) \trianglelefteq P_{\lambda_2}(I)$. This condition follows the well known causal principal in scale space theory: increasing the scale of observation should not create new information. In our framework, increasing the scale should not create new contours. Therefore, all the contours present at a given scale should also be present at finer scales.

If P_λ is an unbiased multi-scale segmentation, $P_\lambda(I)$ increases according to λ and the set $H^* = \bigcup_{\lambda \in \mathbb{R}+} P_\lambda(I)$ defines a hierarchy as an union of nested partitions. The restriction by Guigues of the research space to a hierarchy may thus be justified by the fact that the set of optimal partitions describes a hierarchy. Note that the set $\mathbb{P}$ of partitions on I being finite, H must be also finite.

The family of energies considered by Guigues corresponds to the set of Affine Separable Energies (ASE) which can be written for any partition P of I in n regions

$\{R_1, \ldots, R_n\}$ as:

$$E(P) = D(P) + \lambda C(P) = \sum_{i=1}^{n} D(R_i) + \lambda \sum_{i=1}^{n} C(R_i) = \sum_{i=1}^{n} D(R_i) + \lambda C(R_i)$$

Let us consider a hierarchy H and the sequence $(C^*_\lambda(H))_{\lambda\in\mathbb{R}+}$ of optimal cuts within H. The approach of Guigues is based on the following result: If $E_\lambda(P)$ is an ASE and if $C_\lambda(P)$ is decreasing within $\mathbb{P}$:

$$\forall(P,Q) \in \mathbb{P} \quad P \triangleleft Q \Rightarrow C(P) > C(P)$$

then the sequence $(C^*_\lambda(H))_{\lambda\in\mathbb{R}+}$ is an unbiased multi-scale segmentation. The union of all $(C^*_\lambda(H))_{\lambda\in\mathbb{R}+}$ defines thus a new hierarchy within H. The tree corresponding to the hierarchical structure of $\bigcup_{\lambda\in\mathbb{R}+} C^*_\lambda(H)$ may be deduced from H by merging with their fathers all the nodes which do not belong to any optimal cuts. Note that an equivalent result may be obtained if no condition is imposed to C but if D is increasing according to λ.

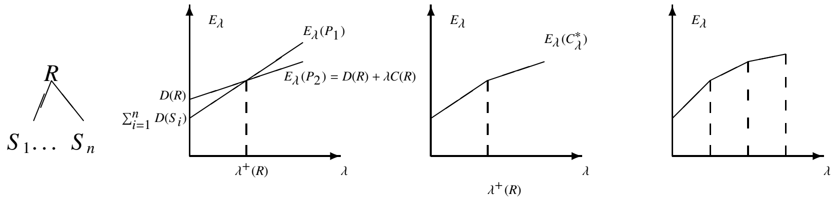
Given a partition $P \in \mathbb{P}$, the decrease of C may be equivalently expressed as a sub-additivity relationship:

$$\forall(R,R') \in P \mid R \text{ is adjacent to } R' \quad C(R \cup R') < C(R) + C(R') \tag{1}$$

Note that the sub-additivity of the regularising term C in common is many applications. For example, if C is proportional to some quantity summed up along contours, C is sub additive due to the removal of the common boundaries between the two merged regions.

Given a hierarchy H, the sequence of optimal cuts $C^*_\lambda(H)$ within H has to be computed. Let us consider one region R at the second level of the hierarchy (computed from the base) and its set of sons $S_1, \ldots, S_n$. Let us additionally consider the tree $H(R)$ rooted at R within H (Fig. 1(a)). Since R is a level 2 node, the hierarchy $H(R)$ allows only two cuts: One encoding the partition P_1 made of the sons of R whose energy is equal to $E_\lambda(P_1) = \sum_{i=1}^n D(S_i) + \lambda \sum_{i=1}^n C(R_i)$ and one encoding the partition P_2 reduced to the single region R. The energy of P_2 is equal to $E_\lambda(P_2) = D(R) + \lambda C(R)$. Due to the sub additivity of C we have $\sum_{i=1}^n C(R_i) > C(R)$. Therefore, using the linear expression of $E_\lambda(P_1)$ and $E_\lambda(P_2)$ in λ, if $\sum_{i=1}^n D(S_i) < D(R)$ the line $E_\lambda(P_1) = \sum_{i=1}^n D(S_i) + \lambda \sum_{i=1}^n C(R_i)$ is below the line $E_\lambda(P_2) = D(R) + \lambda C(R)$ until a value $\lambda^+(R)$ of λ for which the two lines cross(Fig. 1(b)). If $\sum_{i=1}^n D(S_i) \geq D(R)$, $E_\lambda(P_2)$ is always greater or equal to $E_\lambda(P_1)$ in which case we set $\lambda^+(R)$ to 0. Therefore, in both cases the partition P_1 is associated to a lower energy than P_2 for $\lambda = 0$ until $\lambda = \lambda^+(R)$. Above this value the partition P_2 is associated to the lowest energy. In terms of optimal cuts, P_1 corresponds to the optimal cut of $H(R)$ until $\lambda^+(R)$ and P_2 is the optimal cut above this value(Fig. 1(c)). The value $\lambda^+(R)$ is called the *scale of appearance* of the region R.

Guigues shown that the above process may be generalised to the whole tree. Each node of H is then valuated by a scale of appearance. Some of the nodes of H may get a greater scale of appearance than their father. Such nodes do not belong to any optimal cut and are removed from H during a cleaning step which merges them with their fathers. Each node R of the resulting hierarchy belongs to an optimal cut from



(a) H(R) (b) $E_\lambda(P_1), E_\lambda(P_2)$ (c) $E_\lambda(C^*_\lambda(H(R)))$ (d) $E_\lambda(C^*_\lambda(H))$

Fig. 1. (a) a node R of the hierarchy whose sons $\{S_1, \dots, S_n\}$ correspond to initial regions. (b) The energies of the partitions associated to R and $\{S_1, \dots, S_n\}$ plotted as functions of λ. (c) the energy of the optimal cuts within $H(R)$ (a). (d) an example of concave piecewise linear function encoding the energy of the optimal cuts within a global hierarchy H.

$\lambda = \lambda^+(R)$ until the scale of appearance of its father $\lambda^+(\mathcal{F}(R))$, where $\mathcal{F}(R)$ denotes the father of R in H. The value $\lambda^+(R)$ may be set for each node of the tree using a bottom up process. The optimal cut $C^*_\lambda(H)$ for a given value of λ may then be determined using a top-down process which selects in each branch of the tree the first node with a scale of appearance lower than λ. The set of selected nodes constitutes a cut of H which is optimal by construction according to E_λ. The function $E_\lambda(P)$ corresponds to a concave piecewise linear function whose each linear interval corresponds to the energy of an optimal cut (Fig. 1(d)).

Guigues proposed to build a hierarchy H by using an initial partition P_0 and a strategy called the *scale climbing*. This strategy merges at each step the two adjacent regions R and R' such that:

$$\lambda^+(R \cup R') = \frac{D(R \cup R') - D(R) - D(R')}{C(R) + C(R') - C(R \cup R')} = \min_{(R_1,R_2)\in P^2, R_1 \sim R_2} \frac{D(R_1 \cup R_2) - D(R_1) - D(R_2)}{C(R_1) + C(R_2) - C(R_1 \cup R_2)} \tag{2}$$

where P denotes the current partition and $R_1 \sim R_2$ indicates that R_1 and R_2 are adjacent in P.

This process merges thus at each step the two regions whose union would appear at the lowest scale. Such a construction scheme is coherent with the further processes applied on the hierarchy. However, the resulting partition is generally not equal to the hierarchy of optimal cuts $\bigcup_{\lambda\in\mathbb{R}+} P_\lambda(I)$. We show in the next section that other construction schemes of the hierarchy may lead to lower energies.

3 Construction of the initial hierarchy

Many energies have been designed in order to encode different types of homogeneity criteria (piecewise constant [3, 6], linear or Polynomial [3] variations,...). This paper being devoted to the construction schemes of the initial partition, we restrict our topic to the piecewise constant model described by Leclerc [3] and Mumford and Shah [6].

The energy of this model may be written as:

$$E_\lambda(P) = D(P) + \lambda C(P) = \sum_{i=1}^{n} SE(R_i) + \lambda|\delta(R_i)| \tag{3}$$

where $P = \{R_1, \ldots, R_n\}$ represents the partition of the image, $SE(R_i) = \sum_{p\in R} \|c_p - \mu_R\|^2$ is the squared error of region R_i and $|\delta(R_i)|$ is the total length of its boundaries.

Within the Minimum Description Length framework, $SE(R_i)$ may be understood as the amount of information required to encode the deviation of the data against the model, while $|\delta(R_i)|$ is proportional to the amount of information required to encode the shape of the model. Within the statistical framework, the squared error may also be understood as the log of the probability that the region satisfies the model (i.e. is constant) using a Gaussian assumption while $|\delta(R_i)|$ is a regularising term.

Our approach follows the scale climbing strategy proposed by Guigues (equation 2). Given a set W of regions within a partition P we thus consider the scale of appearance of the region R defined as the union of the regions in W. The heuristics below use this basic approach but differ on the sets W which are considered and on the ordering of the merge operations.

3.1 Sequential Merging

Given a current partition P, let us consider for each region R of P, its set $V(R)$ of adjacent regions and the set $\mathcal{P}(V(R))$ of all possible subsets of $V(R)$. Each subset $W \in \mathcal{P}(V(R))$ encodes a possible merging of the region R with at least one of its neighbours. Let us denote by $R^W = \bigcup_{R'\in W} R'$ the region formed by the union of the regions in W. Let us additionally consider the two partitions of R^W: $P_{R^W} = \{R^W\}$ and $P_W = W$. The energies associated to these partitions are respectively equal to $E_\lambda(P_{R^W}) = D(R^W) + \lambda C(R^W)$ and:

$$E_\lambda(P_W) = D(W) + \lambda C(W) = \sum_{R'\in W} D(R') + \lambda \sum_{R'\in W} C(R')$$

where $D(W)$ and $C(W)$ denote respectively the fit to data and the regularising terms of the partition P_W.

Since C is sub additive (equation 1) we have $C(W) > C(R^W)$. The energy $E_\lambda(P_W)$ is thus lower than $E_\lambda(P_{R^W})$ until a value $\lambda^+(R^W)$ called the scale of appearance of R^W (Section 2). Using the scale climbing principle, our sequential merging algorithm computes for each region R of the partition the minimal scale of appearance of a region R^W:

$$\lambda^+_{min}(R) = \arg min_{W\in\mathcal{P}(V(R))} \frac{D(R^W) - D(W)}{C(W) - C(R^W)}$$

the set $W \in \mathcal{P}V(R)$ which realises the min is denoted $W_{min}(R)$.

Given the quantities $\lambda^+_{min}(R)$ and $W_{min}(R)$, our sequential algorithm iterates the following steps:

1. Let P denotes the current partition initialised with an initial partition P_0,
2. For each region R of P compute $\lambda^+_{min}(R)$ and $W_{min}(R)$

3. Compute $R_{min} = \arg min_{R \in P} \lambda^+_{min}(R)$ and merge all the regions of $W_{min}(R_{min})$.
4. If more than one region remains go to step 2,
5. Output the final hierarchy H encoding the sequence of merge operations.

This algorithm performs thus one merge operation at each step of the algorithm. Note that all the regions of $W_{min}(R_{min})$ are adjacent to R_{min}. Therefore, within the irregular pyramid framework, the merge operation may be encoded by a contraction kernel of depth one composed of a single tree whose root is equal to R_{min}. The computation of $\lambda^+_{min}(R)$ for each region R of the partition requires to traverse $\mathcal{P}(V(R))$ whose cardinal is equal to $2^{|V(R)|}$. Therefore, if the partition is encoded by a graph $G = (V, E)$, the complexity of each step of our algorithm is bounded by $O(|V|2^k)$ where $|V|$ denotes the number of vertices (i.e. the number of regions) and k represents the maximal vertices's degree of G. The cardinal of V is decreased by $|W_{min}(R_{min})| - 1$ at each iteration. Since $|W_{min}(R_{min})|$ is at least equal to 2, the cardinal of V decreases by at least 1 at each iteration. The computation of $\lambda^+_{min}(R)$ for each region R of the partition may induce important execution times when the degree of the vertices of the graph is important. However, experiments presented in Section 4 show that the cardinal of the subsets $W \in \mathcal{P}(R)$ may be bounded without altering significantly the energy of the optimal cuts. Let us finally note that this algorithm includes the scale climbing approach proposed by Guigues. Indeed, the merge operations studied by Guigues (Section 2) correspond to the subsets $W \in \mathcal{P}(V(R))$ with $|W| = 2$ which are considered by our algorithm.

3.2 Parallel Merge algorithm

Our parallel merge algorithm is based on the notion of maximal matching. A set of edges M of a graph $G = (V, E)$ is called a maximal matching if each vertex of G is incident to at most one edge of M and if M is maximal according to this property. Moreover, we would like to design a maximal matching M such that the scale of appearance of the regions produced by the contraction of M are as low as possible. Let us denote by $\iota(e)$, the two vertices incident to e. Using the same approach as in Section 3.1 we associate to each edge e of the graph the scale of appearance $\lambda^+(\iota(e))$ (equation 2) of the region $R^{\iota(e)}$ defined as the union of the regions encoded by the vertices incident to e. Following, the same approach as Haxhimusa [7] we define our maximal matching as a Maximal Independent Set on the set of edges of the graph. The iterative process which builds the maximal independent set selects at each step edges whose scale of appearance is locally minimal. This process may be formulated thanks to two boolean variables p and q attached to each edge such that:

$$\begin{cases} p_e^1 = \lambda^+(e) = min_{e' \in \Gamma(e)}\{\lambda^+(e')\} \\ q_e^1 = \bigwedge_{e' \in \Gamma(e)} \overline{p_{e'}^1} \end{cases} \text{and} \begin{cases} p_e^{k+1} = p_e^k \vee \left(q_e^k \wedge \lambda^+(e) = min_{e' \in \Gamma(e) \mid q_{e'}^k}\{\lambda^+(e')\}\right) \\ q_e^{k+1} = \bigwedge_{e' \in \Gamma(e)} \overline{p_{e'}^{k+1}} \end{cases} \quad (4)$$

where $\Gamma(e)$ denotes the neighbourhood of the edge e and is defined as $\Gamma(e) = \{e\} \cup \{e' \in E | \iota(e) \cap \iota(e') \neq \emptyset\}$.

This iterative process stops when no change occurs between two iterations. If n denotes the final iteration, the set of edges such that p_e^n is true defines a maximal matching [7] M which encodes the set of edges to be contracted. Moreover, the set of selected edges corresponds to local minima according to the scale of appearance $\lambda^+(e)$. Roughly speaking if $\lambda^+(e)$ is understood as a merge score, one edge between two vertices will be marked ($p_e^k = true$) at iteration k, if among all the remaining possible merge operations involving these two vertices, the one involving them is the one with the best merge score. Note that the construction of a maximal matching is only the first step of the method of Haxhimusa which completes this maximal matching in order to get a decimation ratio of order 2. The restriction of our method to a maximal matching allows to restrict the merge operations to edges which become locally optimal at a given iteration. We thus favour the energy criterion against the reduction factor. As shown by Bield [8], the reduction factor in terms of edges induced by the use of a maximal matching is a least equal to $2\frac{k-1}{2k-1}$ where k is the maximal vertex's degree of the graph. The edge's decimation ratio may thus be very low for graphs with important vertices's degrees. Nevertheless, experiments performed on 100 natural images of the Berkeley database[1] shown that the mean vertex's decimation ratio between levels on this database is equal to 1.73 which is comparable to the 2.0 decimation ratio obtained by Haxhimusa.

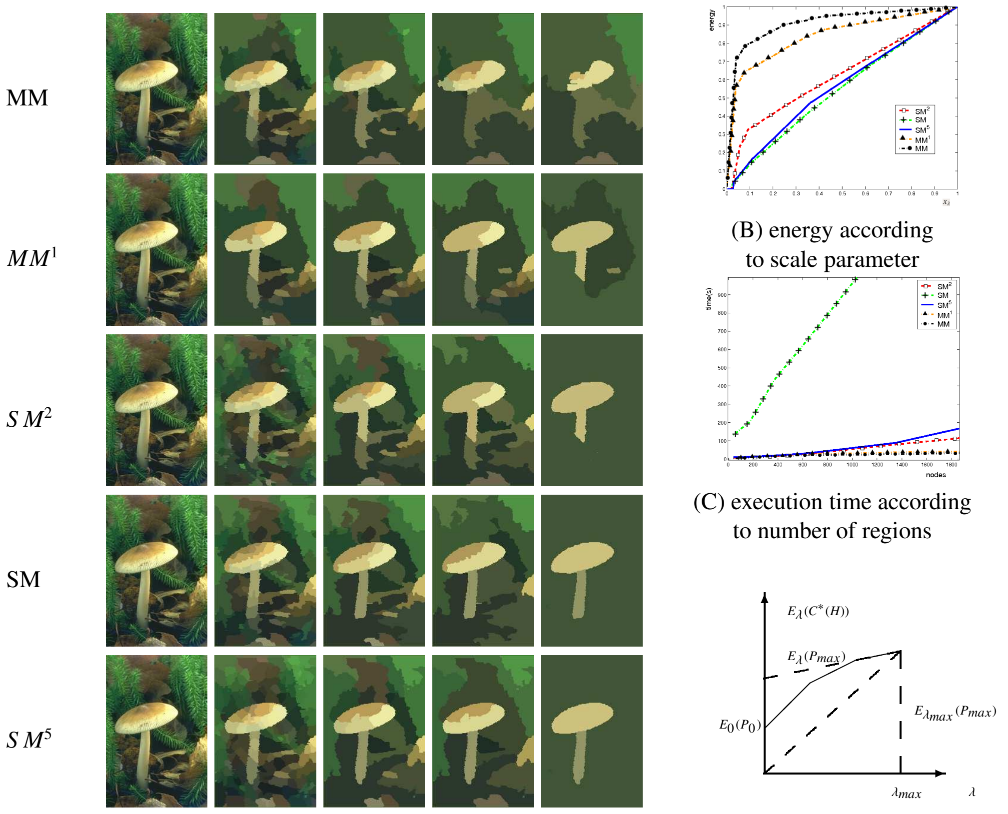
The local minima selected in equation 4 are computed on decreasing sets along the iterations in order to complete the maximal matching. We can thus consider that the detected minima are less and less significants as the iterations progress. We thus propose an alternative solution which consists in contracting at each step only the edges selected at the first iteration ($p_e^1 = true$). These edges correspond to minima computed on the whole neighbourhood of each edge. This method may be understood as a combination of the method proposed by Haxhimusa [7] and the stochastic decimation process of Jolion [9] which consists in merging immediately vertices corresponding to local minima.

4 Experiments

The different heuristics presented in this paper have been evaluated on the Berkeley database. The evaluated heuristics include our parallel merge heuristic based on a maximal matching (MM) and the variation of this method(MM^1) which merges at each step the edges selected during the first iteration (Section 3.2). We also evaluated our sequential method (SM) and two variations of this method: The first variation (SM^2), considers for each region R of the partition the subsets of cardinal 2 of $V(R)$. This method corresponds to the heuristic proposed by Guigues. We also evaluated an intermediate method (SM^5) which restricts the cardinal of the subsets of $V(R)$ to an upper threshold fixed to five in these experiments. All the experiments have used an initial partition obtained by a Watershed algorithm [10].

Given a hierarchy H, since $C^*_\lambda(H)$ is an unbiased multi-scale segmentation (Section 2), the hierarchy H obtained by each of our methods may be associated to a value λ^H_{max} above which the optimal partition P_{max} is reduced to a single region encoding the whole image. The energy of P_{max} is defined as: $E_\lambda(P_{max}) = D_I + \lambda C_I$ where $D_I = SE(I)$ denotes the global image's squared error and $C_I = |\delta(I)|$ the perimeter of the image. Moreover, since the energy of the optimal cuts $E_\lambda(C^*_\lambda(H))$ is a piecewise linear concave function of λ, the function $E_\lambda(C^*_\lambda(H))$ is below the energy $E_\lambda(P_{max})$

[1] available at http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/



(A)mushroom's partitions.

(B) energy according to scale parameter

(C) execution time according to number of regions

(D) energy's bounds of optimal cuts.

Fig. 2. (A) partitions of the mushroom image at different scales. Each line of the array corresponds to an heuristic whose acronym is indicated on the first column. (B) mean optimal energies obtained by our heuristics on the Berkeley database. (C) execution times of the different heuristics on the mushroom image (A) using an initial partition with a varying number of regions. (D)bounds of the optimal cut's energies.

associated to the coarser partition(Fig. 2(D)). Moreover, if P_0 denotes the initial partition, the two points $(0, E_0(P_0))$ and $(\lambda_{max}, E_{\lambda_{max}}(P_{max}))$ belong to the curve. Therefore, $E_\lambda(C^*_\lambda(H))$ being concave, it should be above the line connecting these two points. Moreover, the line connecting $(0, 0)$ to $(\lambda_{max}, E_{\lambda_{max}}(P_{max}))$ being below the line joining $(0, E_0(P_0))$ and $(\lambda_{max}, E_{\lambda_{max}}(P_{max}))$ we have for any hierarchy H and any scale λ: $\frac{\lambda}{\lambda_{max}} E_{\lambda_{max}}(P_{max}) \leq E_\lambda(C^*_\lambda(H)) \leq E_\lambda(P_{max})$ (Fig. 2(D)). We obtain from this last inequality and after some calculus the following equation:

$$\forall \lambda \in \mathbb{R}+ \quad x_\lambda \leq 1 + \frac{x_\lambda - 1}{1 + x_\lambda E_I} \leq \frac{E_\lambda(C^*_\lambda(H))}{E_\lambda(P_{max})} \leq 1 \text{ with } x_\lambda = \frac{\lambda}{\lambda_{max}} \text{ and } E_I = \frac{\lambda_{max} C_I}{D_I}$$

Therefore, using the normalised energy, $\frac{E_\lambda(C^*_\lambda(H))}{E_\lambda(P_{max})}$ and the normalised scale $x_\lambda = \frac{\lambda}{\lambda_{max}}$, any curve $\frac{E_\lambda(C^*_\lambda(H))}{E_\lambda(P_{max})}$ lies in the upper left part of the unit cube $[0,1]^2$. Note that this result is valid for any hierarchy H and thus any heuristic. This result remain valid for any energy whose term C is sub additive.

Using our piecewise constant model (equation 3), the energy $E_\lambda(P_{max})$ is roughly equal to the squared error of the image for small values of λ and may be interpreted as the global variation of the image. The normalised energy allows thus to reduce the influence of the global variation of the images on the energy and to compare energies computed with a same heuristic but on different images. In order to compare our different heuristics, we used the normalised scales x_λ and the normalised energies $\frac{E_\lambda(C^*_\lambda(H))}{E_\lambda(P_{max})}$ in Fig. 2(A) and (B).

Fig. 2(A) represents 5 optimal cuts obtained for increasing values of λ on one image of the Berkeley database. The hierarchies on which these optimal cuts have been produced by our heuristics (Fig. 2(A), column 1).

Fig. 2(B) allows to compare the performance of each heuristic on the whole Berkeley database. This figure shows the mean value of $\frac{E_\lambda(C^*_\lambda(H))}{E_\lambda(P_{max})}$ computed on all the images of the database for each heuristic and each normalised value of λ.

Fig. 2(C) shows the influence of the number of initial regions on the execution time. This curves have been obtained on the image mushroom with different initial partitions obtained by varying the smoothing parameter of the gradient within our Watershed algorithm.

As shown in Fig 2(B) the energy of the optimal partitions obtained by the heuristic MM^1 (−▲−) is lower than the one obtained by the maximal matching heuristic (− • −). This result is confirmed by Fig. 2(A) (lines MM and MM^1) where the heuristic MM removes more details of the mushroom at a given scale. This result is connected to the greater decimation ratio of the MM heuristic. The MM heuristic merges at each step regions with important scale of appearance without considering regions which may appear at further steps with a lower scale of appearance. The algorithms MM and MM^1 induce equivalent execution times on a sequential machine. The execution times of the method MM^1 (−▲−) are overlayed by the execution times of the method MM (− • −) in Fig. 2(C) due to the vertical scale of this figure.

The partitions of the mushroom obtained by the heuristics MM^1 and SM^2 (Fig. 2(A)) seems roughly similar. We can notice that the heuristic MM^1 seems to produce coarser partition at a given scale. However, considering Fig. 2(B), the optimal energy obtained by the heuristic SM^2 (−□−) are lower than the one obtained by MM^1 (−▲−). Note that the heuristic MM^1 produces lower execution times than SM^2 even on a sequential machine(Fig. 2(C)).

As shown by Fig. 2(B) the optimal energies produced by the heuristic SM (− + −) are always below the one produced by the heuristic SM^2 (−□−). Note that, the curve (− + −) is close to the diagonal of the square $[O,1]^2$. This last point indicates that on most of the image of the Berkeley database the hierarchies produced by the SM heuristic are close to the global optimum $\bigcup_{\lambda\in\mathbb{R}+} P_\lambda(I)$ (Section 2). This result is confirmed by Fig. 2(A) where the heuristic SM preserves more details of the image at each scale. However, the heuristic SM is the one which requires the more important execution times on a sequential machine (Fig. 2(C)).

The heuristic SM^5 may be understood as a compromise between SM^2 and SM. As shown by Fig. 2(B) the optimal energies obtained by the heuristic SM^5 (▬) are close to the one obtain by $SM(-+-)$ and below the one obtained by $SM^2(-\square-)$. Moreover, as shown by Fig. 2(C), the execution times required by SM^5 are between the one required by the heuristics SM^2 and SM. Finally, the partitions obtained by the SM^5 heuristic in Fig. 2(A) are close to the one obtained by the heuristic SM.

5 Conclusion

The Scale Set framework is based on two steps: the determination of a hierarchy according to an energy criterion and the determination of optimal cuts within this hierarchy. We have presented in this article parallel and sequential heuristics to build such hierarchies. Our sequential heuristics provide hierarchies which are close from the optimal hierarchy encoding the whole sequence of optimal partitions. These methods may however require important execution times. We thus proposed an heuristic performing a compromise between the search of the global energy minima and the execution times.

Our parallel methods provide greater energies than the one produced by Guigues's heuristic. However, these methods require less execution times even on sequential machine.

In the future, we would like to study in detail the behaviour of the different energies, associated to each regions, in order to perform cross-level regions matching within hierarchies of optimal cuts.

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